RPERESENTATION THEORY MIDTERM EXAMINATION

If you need to use a theorem proved in class to solve your problem, you may quote it without proof. Please note that this is a two hour exam (which starts at 10:45 am).

(1) Let $G = C_2 = \{1, g | g^2 = 1\}$ be the cyclic group with two elements. Let k be a field of characteristic 2, and let V be a two dimensional vector space over k with basis v, w. Define a representation $\rho : C_2 \to Aut_k(V)$ of C_2 on V such that $\rho(g)(v) = w$ and $\rho(g)(w) = v$. Is ρ an indecomposable representation? Is ρ an irreducible representation? (8 marks)

(2) Let G be a finite abelian group. Prove that any irreducible representation of G over $\mathbb{C}$ is one dimensional. (8 marks)

(3) Let $\rho : S_3 \to GL_2(\mathbb{C})$ denote the standard representation of S_3 on $\mathbb{C}^2$. Define a new representation $\psi : S_3 \to GL_2(\mathbb{C})$ by $\psi(\sigma) = sgn(\sigma)\rho(\sigma)$ (here $sgn(\sigma)$ is 1 or -1 depending on whether σ is an even or odd permutation respectively). Show that ρ and ψ are equivalent representations via an explicit isomorphism. (9 marks)

(4) Let G be a finite group which has an abelian normal subgroup of index m. Show that any irreducible representation of G over $\mathbb{C}$ has dimension at most m. (10 marks)

Date: February 26, 2014.